2020
MATHEMATICS
[GENERAL]
Paper : III

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.

Symbols have their usual meanings.

GROUP–A
(Linear Programming and Game Theory)
[Marks : 50]

1. Answer any **four** questions: 1×4=4

a) Define feasible solution to a L.P.P.

b) Define convex polyhedron.

c) Write down the mathematical form of general L.P.P.

d) Do the vectors (4, 3, 2), (2, 1, 4), (2, 3, –8) form a basis for E^3?

e) Define convex hull. Give an example.

f) Find the extreme points, if any of the set

$$X = \{(x, y)/x^2 + y^2 \le 25\}.$$

2. Answer any **six** questions: 2×6=12

a) Express (2, 4, –3) as a linear combination of (1, 3, 1) and (0, 2, 5).

b) Find the solution of the equations:

$$2x_1+3x_2+x_3=8$$
$$x_1+2x_2+2x_3=5$$

c) Verify whether the set of vectors form a spanning set for E^3;

(1, –1, 0), (0, 0, 1), (1, 1, 0).

d) What is the convex hull of the set

$$X = \left\{(x, y)\Big/\frac{x^2}{3} + \frac{y^2}{2} = 1\right\}?$$

e) Reduce the following problem to standard maximinization form:

Maximize $Z = 2x_1 + x_2$

subject to $x_1 \le 4$

$2x_1 + x_2 \ge 1, \quad x_1, x_2 \ge 0$

f) Find the dual of the following L.P.P.:

Maximize $Z = -x_1 + 3x_2$

subject to $2x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \leq 5$

$x_1 + 6x_2 \leq 9$

$x_1, \ x_2 \geq 0$

g) Write the rule for determining saddle point.

h) Using maximini-minimax principle solve the following game:

$$A \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix}$$

3. Answer any **four** questions: 6×4=24

a) Find a B.F.S. of the following system of equations:

$x_1 + 4x_2 - x_3 = 3$

$5x_1 + 2x_2 + 3x_3 = 4$

b) Solve the following L.P.P. graphically:

Minimize $Z = x_1 + 7x_2$

subject to $-x_1 + 2x_2 \leq 8$

$x_1 - x_2 \leq 4$

$x_1, \ x_2 \geq 0$

c) Solve by simplex method:

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$

$x_1, \ x_2, \ x_3 \geq 0$

d) Find the optimal solution of the following L.P.P. solving its dual:

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

subject to $x_1 + x_3 \geq 2$

$x_2 + x_3 \geq 5$

$x_1, \ x_2, \ x_3 \geq 0$

e) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	10	9	8	8
O_2	10	7	10	7
O_3	11	9	7	9
O_4	12	14	10	4
b_j	10	10	8	

f) Use dominance property to reduce the following pay-off matrix and solve the game:

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	–5	3	1	15
	A_2	5	5	4	6
	A_3	–4	–2	0	–5

4. Answer any **one** question: 10×1=10

a) Solve the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	50	30	220	1
O_2	90	45	170	3
O_3	250	200	50	4
b_j	4	2	2	

b) Transform to LPP and hence solve the game problem whose pay-off matrix is

$$\begin{bmatrix} 2 & -3 & 4 \\ -3 & 4 & -5 \\ 4 & -5 & 6 \end{bmatrix}$$

GROUP–B

(Probability Theory)

[Marks : 30]

5. Answer any **four** questions: 1×4=4

a) Give the classical definition of probability.

b) Define conditional probability.

c) State the Bayes' theorem on conditional probability.

d) Give the definition of k-th central moment of a random variable X.

e) Define the variance of a random variable.

f) Give examples of two continuous probability distributions.

6. Answer any **four** questions: 2×4=8

a) If A, B, C are any three events, then prove that

$$P(A+B+C) = P(A)+P(B)+P(C)-P(AB)$$
$$-P(BC)-P(CA)+P(ABC).$$

b) In any random experiment E, if A and B are any two events, then show that

$$P(AB) = P(A).P(B/A) = P(B).P(A/B).$$

c) Two dice are thrown simultaneously. Find the probability of getting a total of 4 points in a single throw.

d) Show that

$$P(X.Y)>0 \Rightarrow E(XY)>E(X)E(Y).$$

e) Prove that the probability distribution function is rightly continuous.

f) Find the parameters of binomial variate X, whose mean and variance are $\frac{15}{2}, \frac{15}{4}$.

7. Answer any **three** questions: 6×3=18

a) The probability of a man hitting a target is $\frac{1}{3}$. How many times must he fire so that the probability of hitting the target atleast once is more than 90%?

b) Deduce Poisson distribution from Binomial distribution. Hence obtain mean of Poisson distribution.

c) Find k such that

$$f(x)=\begin{cases} 0 & \text{if} \quad x<0 \\ kx(1-x) & \text{if} \quad 0 \le x < 1 \\ kx^2 & \text{if} \quad 1 \le x \le 2 \\ 0 & \text{if} \quad x>2 \end{cases}$$

is a probability distribution function. Also obtain the distribution function.

d) Find the n-th moment of the normal distribution N(m.m) about the mean m. $\left[\text{Given that } \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}\right]$.

e) i) Show that if X and Y are independent then $E(XY)=E(X).E(Y)$ for continuous distribution.

ii) Define mean, covariance and correlation coefficient for two-dimensional random variables X and Y.

GROUP–C

(Statistics)

[Marks : 20]

8. Answer any **four** questions: 1×4=4

a) Define a Ogive Curve.

b) Define the median of a distribution.

c) What is the relation between mean, median and mode in case of a symmetrical distribution?

d) What is a Quartile deviation?

e) Define 'skewness' of a distribution.

f) Show that the mean deviation about mean is always zero.

9. Answer any **three** questions: 2×3=6

a) For the given set of data {3, 2, 5, 7}, show that G.M.>H.M.

b) Show that the sum of deviation of the sample x_1, x_2, ..., x_n of size n is zero.

c) Calculate the variance and sd of {3, 4, 8}.

d) Define Quartiles of a distribution.

e) State two important properties of regression co-efficient.

10. Answer any **two** questions: 5×2=10

a) Draw a pie chart of the following data:

Year	1998	1999	2000	2001	2002
No. of tourist at a place (in thousand)	14	17	20	22	29

b) Show that the A.M. of two regression coefficients is always greater than or equal to the correlation coefficient.

c) Find the correlation coefficient from the following data:

x	2	3	4	5	6
y	5	2	3	4	1

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