2021

MATHEMATICS

[GENERAL]

Paper : II

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

GROUP–A

(Classical, Abstract and Linear Algebra)

[Marks : 50]

1. Answer any **two** questions: 1×2=2

   a) If $A=\begin{bmatrix}2 & 5\\ 3 & 1\end{bmatrix}$, find $A^2-3A-13I$.

   b) Express 1+i in the form of $r(\cos\theta+i\sin\theta)$

   c) Give an example of symmetric relation.

   d) Find amplitude of $-1-i$.

2. Answer any **five** questions: 2×5=10

   a) Find the remainder when $f(x)=x^3+5x^2+7x+2$ is divided by $x-1$.

   b) If $A=\begin{bmatrix}1 & 2 & 3\\ 6 & 7 & 8\\ 6 & -3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 2\\ 5 & 6 & 1\end{bmatrix}$, verify $(AB)^T=B^TA^T$.

   c) Prove that amp (z_1,z_2)=amp(z_1) +amp(z_2)

   d) Determine the number of positive and negative real roots of the education

   $$x^5+4x^4-3x^2+x-6=0$$

   e) If the function $f:R\rightarrow R$ be defined by $f(x)=x^2+1$? Find $f^{-1}(17)$.

   f) Without expending find the value of

   $$\begin{vmatrix}17 & 58 & 97\\ 19 & 60 & 99\\ 18 & 59 & 98\end{vmatrix}.$$

   g) If the roots of the equation $x^3-px^2+qx-r=0$ are in G.P, show that $q^3=p^3r$.

3. Answer any **three** questions: 6×3=18

   a) Prove that if R bearing with unity element 1, then this is the unique multiplicative identity.

b) If $z_r = \cos\frac{\pi}{3^r} + \sin\frac{\pi}{3^r}$. Prove that

$z_1, z_2, z_3, \ldots \infty = i$.

c) Prove that the set G with an operation $*$, which is defined by $x * y = \frac{x+y}{xy+1}$, forms an Abelian group.

d) Prove that in a group $(G, *)$ the equations $a * x = b$ and $y * a = b$ have unique solutions.

e) Solve $x^3 – 6x – 9 = 0$ by Cardons method.

4. Answer any **two** questions: 10×2=20

a) i) Show that the group given by the following table is cycle

*	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a.

ii) Show that the vectors (1,0,0), (0,1,0), (0,0,1) and (1,2,3) generate the same space as generated by the vectors (1,0,0), (0,1,0), (0,0,1).

b) Find the eigenvalues and corresponding eigenvector of the matrix

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$$

c) i) Find the rank of the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}.$$

ii) Prove that if two rows or two columns of a determinant are identical then the value of the determinant is zero.

GROUP–B

(Analytical Geometry and Vector Algebra)

[Marks : 50]

5. Answer any **four** questions: 1×4=4

a) For any vectors $\bar{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\bar{b} = p\hat{i} + q\hat{j} + r\hat{k}$ calculate $|\bar{a} \times \bar{b}|$.

b) Prove that $4x^2 + 9y^2 + 12xy + 4x + 6y + 1 = 0$ represents pair of straight lines.

c) Write perpendicular distance from (x_0, y_0) to $ax + by + c = 0$.

d) If $\bar{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and $\bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$, find $\bar{a}.\bar{b}$.

e) Can the numbers $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ be the direction cosine of a straight line?

f) Transform the equation $x^2-y^2+4x+6y+1=0$ it the once transform into parallel ones passing through the point (2, –1).

6. Answer any **six** questions: 2×6=12

a) When two vectors $\bar{a}$ and $\bar{b}$ are called linearly independant?

b) Find the polar and parametric form of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

c) Find the equation of the line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2).

d) Find the unit vector perpendicular to both $2i - j + 4k$ and $i + j + k$.

e) For any two vectors $\bar{a}$ and $\bar{b}$ if $|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$ prove that $\bar{a}$ and $\bar{b}$ are perpendicular to each other.

f) Find the radius of the circle $x^2+y^2+z^2=25$, $x+2y+2z+9=0$.

g) Whatever be the value α, prove that locus of the intersection of the straight lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ is a circle.

h) If PSP' be the focal chord of the conic $\frac{l}{r} = 1 - e\cos\theta$, show that $\frac{1}{SP}+\frac{1}{SP'}=\frac{2}{l}$, where the rotations have usual meanings?

7. Answer any **four** questions: 6×4=24

a) Show that for any vector $\bar{a}$ can be expressed as $\bar{a} = (\bar{a}.\hat{i})\hat{i} + (\bar{a}.\hat{j})\hat{j} + (\bar{a}.\hat{k})\hat{k}$.

b) If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosine of two perpendicular lines, then show that $(l_1 + l_2)^2 + (m_1 + m_2)^2 + (n_1 + n_2)^2 = 2$.

c) Find the equation of the plane through (2, 1, 0) and perpendicular to $2n - 4y + 3z = 2$ and $x + y + z = 5$.

d) Find the distance of the point (2, 3, –1) from the line $\frac{x-1}{-2} = \frac{y+5}{-1} = \frac{z+15}{2}$.

e) Show that the equation

$$8x^2+8xy-6y^2-2x-11y=3$$

represents a pain of interesting straight line and the angle between them is $\tan^{-1}(\delta)$.

f) If by a rotation of co-ordinate axes the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ show that $a+b=a'+b'$.

8. Answer any **one** question: 10×1=10

a) i) Show that the circle

$x^2+y^2+z^2-3x-y+z=5$,

$x-y-2z=0$ and

$x^2+y^2+z^2+4x+2y+2z=5$,

$x+y+z+1=0$ lie on a common sphere. Hence find the equation of the sphere.

ii) Show that the points (1, 0,1) (2, –1, 2), (3, 4, 5) and (1, –1,–1) are non-coplanar. Hence find the distance of the fourth point from the plane passing through the first three points.

5+5

b) i) Resolve a vector $\overline{r}$ in the direction of three non-coplanar vectors $\overline{a}, \overline{b}, \overline{c}$.

ii) Find the equation of a sphere passing through four non-coplanar points

$(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4)$.

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