U.G. 2nd Semester Examination - 2021

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-02

(Differential Equations)

Full Marks : 30 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **five** questions: 2×5=10

   a) Find the integrating factor of $x \log x \frac{dy}{dx} + y = 2 \log x$

   b) Find the particular solution of the differential equation $y'' + 2y' + y = 3 - 2\sin x$.

   c) Solve: $9yp^2 + 4 = 0$ and examine for singular solutions, where $p \equiv \frac{dy}{dx}$.

   d) Find the differential equation of the circles touching the y-axis at the origin.

   e) Find the particular solution of $\cos y\, dx + (1 + 2e^{-x})\sin y = 0$ when $x = 0, y = \frac{\pi}{4}$.

[Turn Over]

   f) Eliminate the arbitrary function and form the the PDE from $z = xy + f(x^2 + y^2)$.

   g) Solve: $p \tan x + q \tan y = \tan z$.

   h) Find the characteristics of $x^2 r + 2xys + y^2 t = 0$.

2. Answer any **two** questions: 5×2=10

   a) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$.

   b) Find the solution of the initial value problem

   $x^2y'' - xy' - 3y = 0, y(1) = 1, y'(1) = -2$.

   c) Solve the following equation by Charpit's method : $\sqrt{p} + \sqrt{q} = 1$.

   d) If $y = y_1(x)$ be a solution of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$, then prove that the general solution of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$, Where P(x), Q(x), R(x) are continuous functions of x, can be determined in the form $y = v(x)y_1(x)$.

   e) Find a complete integral of $zpq = p + q$.

3. Answer any **one** question: 10×1=10

a) i) Solve :

$$\frac{dx}{dt} + 5x + y = e^t, \quad \frac{dy}{dt} - x + 3y = e^{2t}$$

5

ii) Solve by Lagrange's method,

$$x^2(y-z)\frac{\partial z}{\partial x} + y^2(z-x)\frac{\partial z}{\partial y} = z^2(x-y).$$

5

b) i) Solve:

$$x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x).$$

5

ii) Solve:

$$z(x+y)\frac{\partial z}{\partial x} + z(x-y)\frac{\partial z}{\partial y} = x^2 + y^2.$$ 5

c) i) Solve:

$$\frac{dx}{y^2+yz+z^2} = \frac{dy}{z^2+zx+x^2} = \frac{dz}{x^2+xy+y^2}.$$ 5

ii) Reduce the differential equations $y = 2px - p^2y, p = \frac{dy}{dx}$ to Clairaut's form by the substitution $y^2 = Y, x = X$ and then obtain the complete primitive and singular solution, if any. 5

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