U.G. 4th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Generic Elective(GE)

Course Code : MATH-H-GE-T-02

Full Marks : 30 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **five** questions: 2×5=10

a) If the integrating factor of the differential equation $\frac{dy}{dx}+P(x)y=g(x)$ is x^3 find the coefficient function *P(x)*.

b) Find the number of linearly independent solution of the form x^r corresponding to the equation $(x^3D^3-6xD+12)y=0$.

c) Find the differential equation of the family of all right circular cones whose axis coincides with z-axis.

d) Eliminate the arbitrary function f from $z=f\left(\frac{xy}{z}\right)$.

e) Find the general solution of the equation $p^3-4xyp+8y^2=0$.

f) Find the singular solution of $8ap^3=27y$.

g) Show that the curve for which the normal at every point passes through a fixed point is a circle.

h) Solve: $\left(\frac{d^3y}{dx^3}\right)^2+x\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}=0$.

2. Answer any **two** questions: 5×2=10

a) Show that $x^{-\frac{1}{2}}cosx$ and $x^{-\frac{1}{2}}sinx$ be two linearly independent solutions of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$ and find the general solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=x^{-\frac{3}{2}}$. 5

b) Find the value of y_0 for which the solution of the initial value problem $\frac{dy}{dx}-y=7-4sinx, y(0)=y_0$ remains finite as $x\to\infty$. 5

c) Suppose that a fourth order differential equation has a solution $y=-3e^{3x}xsinx$. Find the differential equation, assuming it is homogeneous and has constant coefficients. Hence find the general solution of the 4th order differential equation. 5

d) Solve the differential equation:

$$(2+x)^2\frac{d^2y}{dx^2}+(2+x)\frac{dy}{dx}+4y=2sin(2\log(2+x)).$$ 5

e) Let $f(x,y)$ be differentiable function satisfying the equation $\left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 = xy$. After the change of variables $x = s + t$ and $y = s - t$ what equation $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ will satisfy? 5

3. Answer any **one** question: 10×1=10

a) i) Find the canonical form of the equation $x^3 u_x + 2y^2 u_y = x^2 y$. 5

ii) Solve: $\frac{dx}{cos(x+y)} = \frac{dy}{sin(x+y)} = \frac{dz}{\left(z+\frac{1}{z}\right)}$. 5

b) i) Solve the differential equation

$$\left(2 + 3x\sqrt{x^2 - y^2}\right)dx + \left(-5 - 3y\sqrt{x^2 - y^2}\right)dy = 0.$$

5

ii) Solve the following system of simultaneous linear differential equation:

$$(D + 5)x + y = e^t$$
$$(D + 3)y - x = e^{2t}$$

5

c) i) Find the value of a and the solution of the initial value problem

$$\frac{d^2y}{dx^2} - 9y = e^{-x}, y(0) = -1, \left(\frac{dy}{dx}\right)_{at\ x=0} = a$$

and y approaches 0 as $x \to \infty$. 5

ii) Solve the partial differential equation $p - 3x^2 = q^2 - y$ by Charpit's method. 5

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