U.G. 5th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-11

(Partial Differential Equations & Applications)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **ten** questions from the following: 2×10=20

   i) Form the partial differential equation by eliminating the constants a and b from $z = ax + by + ab$.

   ii) When a PDE is called well-posed? Give an example.

   iii) Verify whether the partial differential equations $xp - yq = x$ and $x^2 p + q = xz$ are compatible.

   iv) Find the general integral of the equation $y^2 p - xyq = x(z - 2y)$.

   v) State mean value theorem for harmonic functions.

   vi) Solve the Cauchy initial value problem $u_t + cu_x = 0,\ x \in R,\ t > 0$ satisfying $u(x,0) = f(x), x \in R$.

   vii) Find the complete integral of the PDE $pq = 1$.

   viii) Solve the PDE: $(D^3 - 3D^2D' + 4D'^3)u = 0$.

   ix) Eliminate the arbitrary function from $z = xy + f(x^2 + y^2)$, and hence, obtain the corresponding partial differential equation.

   x) Show that along every characteristic strip of the PDE $F(x,y,z,p,q)=0$, the function $F(x,y,z,p,q)$ is constant.

   xi) Show that if a harmonic function vanishes everywhere on the boundary of the domain where the equation is defined, then it is identically zero everywhere.

   xii) Find the characteristic curves of the PDE $u_{xx} + xu_{yy} = 0,\ x < 0$.

   xiii) Show that if the Dirichlet problem for a bounded region has a solution, then it is unique.

   xiv) Find the particular integral of $(D - D' - 1)(D - D' - 2)u = x$.

xv) Let $\{f_n\}$ be a sequence of functions, each of which is continuous on $\bar{R}$ and harmonic on R. If the sequence $\{f_n\}$ converges uniformly on boundary of R, then it uniformly converges on $\bar{R}$.

2. Answer any **four** questions: 5×4=20

i) Examine whether the equation $(y-z)dx+(z-x)dy+(x-y)dz=0$ is integrable and if so, obtain its integral.

ii) Find the complete integral of $px+3qy=2(z-x^2q^2)$ by Charpit's method.

iii) Solve the following Cauchy problem by the method of characteristics :

$pz+q=1$ with initial data $y=x, z=\frac{x}{2}$.

iv) Reduce the equation

$z_{xx}-6z_{xy}+13z_{yy}+6z_x-18z_y+12=0$

to canonical form.

v) Obtain the solution of

$(D_x+2D_y^2)z=0$

by the method of separation of variables.

vi) Deduce Green's first and second identity.

3. Answer any **two** questions: 10×2=20

i) Find the solution of the equation

$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}, 0<x<a,\ t>0,\ k>0$

subject to the conditions

a) $u(x,t)$ is bounded as $t\to\infty$

b) $\frac{\partial u}{\partial x}=0$ at $x=0$ and $x=a$ for $t\geq 0$

c) $u(x,0)=x(a-x), 0\leq x\leq a.$

ii) The ends of a thin uniform string of length l are fixed and its mid-point is pulled aside transversely through a distance h and is then released from rest. Determine the subsequent motion.

iii) Obtain the solution of the Dirichlet's problem

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, 0<x<1, \quad 0<y<1$

subject to the boundary conditions

a) $u(0,y)=0, u(1,y)=0, 0\leq y\leq 1$

b) $u(x,0)=0, u(x,1)=x(1-x).$

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