U.G. 5th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-12

Course Title : Group Theory - II

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

GROUP-A

(Marks : 20)

1. Answer any **ten** questions from the following: $2\times10=20$

a) Define group action on a set.

b) Find the class equation of the symmetric group S_3.

c) Let G be a group and $a \in G$. Prove that $a \in Z(G)$ if and only if $Cl(a) = \{a\}$, where $Z(G)$ is the center of G and $Cl(a)$ is the conjugacy class of a.

d) Show that any group of order 14 contains a normal subgroup of order 7.

e) Find the number of distinct conjugacy classes in S_4.

f) Show that any group of order 15 is commutative.

g) Let G be a group. Show that $Inn(G)$ is a normal subgroup of $Aut(G)$.

h) Show that a group of prime order is a simple group.

i) Find Aut($\mathbb{Z}$).

j) Define the commutator subgroup or derived subgroup of a group.

k) Is external direct product of two cyclic groups always cyclic? Justify.

l) Prove that $\mathbb{Z}$ under addition is not isomorphic to $\mathbb{Q}$ under addition.

m) State the Sylow's third theorem.

n) What is the order of the factor group $\frac{\mathbb{Z}_{60}}{<15>}$?

o) Define internal direct product of two subgroups H_1 and H_2 of a group G.

GROUP-B

(Marks : 20)

2. Answer any **four** questions: 5×4=20

a) State and prove the fundamental theorem of Abelian group. 5

b) Show that any group of order $11^2 .13^2$ is Abelian. 5

c) In $\mathbb{Z}_{40} \oplus \mathbb{Z}_{30}$, find two subgroups of order 12. 5

d) Show that any group of prime power order always have a non-trivial center. 5

e) Find two groups G and H such that G and H are not isomorphic but Aut(G) is isomorphic to Aut(H). Show that U(8) is not isomorphic to U(10). 2+3

f) Find Inn(D_4), where Inn(D_4) denotes the group of inner automorphisms of D_4. 5

GROUP-C

(Marks : 20)

3. Answer any **two** questions: 10×2=20

a) i) Let G be a group and H be a normal subgroup of G. Show that the function $G \times H \mapsto H$ defined by $(g, h) \mapsto ghg^{-1}$ is a group action. 2

ii) Let G be a cyclic group of order mn, where m, n are positive integers such that $gcd(m, n) = 1$. Show that $G \cong Z_m \times Z_n$. 4

iii) Find all the Sylow 3-subgroups of S_4. 4

b) i) State Cauchy's theorem. 2

ii) Show that any group of order p^2 is commutative, where p is a prime. 3

iii) Let G be an Abelian group of order n and m be a positive divisor of n, then show that G has a subgroup of order m. 5

c) i) Define simple group. 1

ii) Let p and q be two prime numbers. Then show that no group of order pq is simple. 4

iii) Prove that any group of order 30 is not simple. 5

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