U.G. 5th Semester Examination - 2021

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-11

(Quantum Mechanics & Applications)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

a) The wavefunction of a particle confined to the *x* axis is $\psi = e^{-x}$ for x>0 and $\psi = e^{+x}$ for x<0. Calculate the probability of finding the particle between $x = -1$ and $x = +1$.

b) Write down the expression of angular momentum operator in spherical polar coordinate.

c) Which of the following two functions would make satisfactory wavefunctions for all values of the variable *x*?

i) $\psi = Ne^{-ax^2}/(3-x)$

ii) $\psi = Ne^{-ax^2}$

d) For a particle moving freely along the *x*-axis, show that the Heisenberg uncertainty principle can be written in the alternative form:

$$\Delta\lambda\Delta x \geq \frac{\lambda^2}{4\pi}$$

where $\Delta\lambda$ is the uncertainty in position of the particle and $\Delta\lambda$ is the simultaneous uncertainty in the de Broglie wavelength.

e) Show that the velocity of a particle is represented by the group velocity, not the phase velocity.

f) Write down the radial wave function of the 1st and excited state of Hydrogen atom and plot the function.

g) What is the difference between Zeeman effect and Stark effect?

h) What is space quantization? Explain with a suitable diagram.

2. Answer any **two** questions : 5×2=10

a) i) The time independent wave function of a particle of mass m moving in a potential $V(x) = \alpha^2 x^2$ is

$$\psi(x) = \exp\left(-\sqrt{\frac{m\alpha^2}{2h^2}}x^2\right)$$

α being a constant. Find the energy of the system.

ii) Give a physical example of particle in a box. 1+4

b) i) Determine $\langle E \rangle$ for a particle in a box with wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{L^5}}x(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

ii) Prove that $[L_x, L_y] = i\hbar L_z$, where L_x, L_y and L_z are the components of orbital angular momentum. 3+2

c) i) What are the possible values of L for a system of two electrons whose orbital quantum numbers are l_1=1 and l_2=3?

ii) What are the possible values of S?

iii) What are the possible values of J?

(L, S, J have their usual meaning)

iv) The ground state of chlorine is $^2P_{3/2}$ Find its magnetic moment μ_J under LS-coupling. (1+1+1+2)

d) i) The wavelength (λ) and the frequency (ν) in a wave group are related by $\lambda = \frac{c}{\sqrt{\nu^2 - \nu_0^2}}$. Show that the group velocity, $\nu_g = \frac{c^2}{\nu\lambda}$.

ii) Prove that the eigenvalues of hermitian operators are real. 3+2

3. Answer any **two** questions: 10×2=20

a) i) Prove that if two operators A and B commute, then they have common eigenfunctions.

ii) What is complex vector space?

iii) What is Dirac bra-ket notation? How inner product of state vector is represented using Dirac bra-ket notation? 3+2+(2+3)

b) i) Write the schrodinger equation for many electron system explaining all the terms.

ii) Specify the symmetry of the following function

$$\psi(x_1, x_2) = 4(x_1 - x_2)^2 + \frac{10}{x_1^2 + x_2^2}$$

iii) What is spin orbit interaction energy?

iv) Describe spin orbit coupling in Hydrogen fine structure. 2+2+3+3

c) i) What is space quantization? Explain with a suitable diagram. Discuss briefly spin angular momentum. What is the importance of Stern-Gerlach experiment?

ii) Discuss normal Zeeman effect with energy level diagram. (2+2+1+1)+4

d) i) Discuss briefly the idea of spin magnetic moment.

ii) Write down the time-independent Schrödinger equation for the motion of the electron in hydrogen atom, assuming that the proton is at rest.

Given : $\vec{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$

Separate the Schrödinger equation into one radial and two angular parts. 3+7

———————