U.G. 5th Semester Examination-2021

PHYSICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : PHY-H-DSE-T-01

(Advanced Mathematical Physics-I)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions : 2×5=10

   a) Write down the steps that are followed in the Gram-Schmidt orthogonalization method for converting a linearly independent basis into an orthonormal one.

   b) Write any two properties required for an arbitrary set of n objects to form a linear vector V_n.

   c) Prove that for every vector $|\varphi>$ in V_n, there exists an inverse under addition, $|-\varphi\rangle$ such that $||\varphi>+|-\varphi>=|0>$.

   d) Find the value of $L\left[e^{ax}\right]$, where a is a constant and L represents the Laplace transformation.

   e) State the first and second shifting theorems in connection to Laplace Transformation.

   f) In a Cartesian coordinate system, the distinction between the contravariant and the covariant tensors vanishes. Explain.

   g) What do you mean by a Tensor of zero order? Give an example.

   h) Write down the relationship between alternate and Kronecker tensor.

GROUP-B

Answer any **three** questions: 10×3=30

2. a) Write down the fundamental form and the metric tensor of the Minkowski space in spherical coordinates. Why is Minkowski space not Euclidean?

   b) Calculate the components of the Riemann–Christoffel tensor of the space defined by metric $ds^2=\left(dx^1\right)^2+\left(dx^2\right)^2+\left(dx^3\right)^2-e^{-t}\left(dt\right)^2$.

   c) Write down the elaborate form of inertia tensor for a continuous body in Cartesians.

   d) What are the diagonal and the off-diagonal elements stand for? 3+2+3+2

3. a) Show that the process of contraction of an Nth-order tensor produces another tensor, of order N - 2.

b) State and prove the 'quotient law' for tensors.

c) Show that T_{ij} given by

$$T=\left[T_{ij}\right]=\begin{pmatrix} x_2^2 & -x_1x_2 \\ -x_1x_2 & x_1^2 \end{pmatrix}$$

are the components of a second-order tensor.

3+5+2

4. a) Considering $A=\left[a_{ij}\right], B=\left[b_{ij}\right]$, and that $B=A^{-1}$ show the relationship $\frac{\partial a}{\partial u^k}=ab^{ij}\frac{\partial a_{ij}}{\partial u^k}$ by taking that the determinant a=|A|.

b) If f(x) is a periodic function of period P > 0, that is, if f(x+P) = f(x), then show that

$$L\left[f(x)\right]=\frac{1}{1-e^{-p^{p}}}\int_0^P e^{-px}f(x)\,dx$$

5+5

5. a) State and prove the Laplace transform of the derivative of f(t). Hence show that Laplace transform of derivatives of order n can be represented as

$$L\left[f^n(t)\right]=s^nL\left[f(t)\right]-s^{n-1}f(0)-s^{n-2}f'(0)$$
$$-s^{n-3}f''(0)-\ldots-f^{n-1}(0)$$

b) Solve the initial value problem using Laplace transformations:

$y''+y=0; y(0)=y'(0)=0$ and $f(t)=0$ for $t<0$ but $f(t)=1$ for $t\geq 0$.

c) Let $Y_z=AX_z$ be a linear transformation expressed concerning some original Z-basis $\{Z_1, Z_2, \ldots, Z_n\}$. What is the expression for this same transformation when expressed w.r.t. some arbitrary other W-basis?

4+3+3

6. a) What do you mean by linear mapping or linear transformation? Explain with a suitable example.

b) When do the two linear spaces are said to be homomorphic and isomorphic?

c) Prove that A linear transformation Y = AX is nonsingular if and only if A, the matrix of the transformation, is nonsingular.

d) Given the basis vectors A, B, C, below, use the Gram-Schmidt method to find an orthonormal set of basis vectors e_1, e_2, e_3.

A = (0,0,5,0), B = (2,0,3,0) and C = (7,1,-5,3)

3+2+2+3